CC Math Questions for Dr. Milgram
November 12, 2013 @ 8 pm (EST)

1. Why specifically is the workload so difficult for the younger grades? What is the purpose for overwhelming kindergartners and 1st graders, etc.?

   These standards were written under severe deadline restrictions. The entire process took between 9 and 10 months. As a result, there are many inconsistencies and ill thought out sequences in the material even though the overall mathematical level is quite solid. For example, until the final version, the requirement for kindergartners was counting to 125 while it was only counting to 100 in first grade. Additionally, for reasons that I do not know or understand, the main authors of CC where people who have never written mathematics standards before and, consequently, had no experience and no track record. They were quite capable, but there wasn't time for them to get the best advice on squencing and organization, or for them to learn the relevant research.

2. Why did they lower the standards for high schoolers in most cases from before?

   The best I can do here is an informed conjecture. It could be completely off base but it does seem to explain most of what was going on. There were basically 3 groups involved in overseeing the development of the math standards,Achieve, the Gates Foundation, and Marc Tucker's NCEE. Tucker appears to strongly believe in “efficiency,” which basically means “teach the minimal amount of material that students will *really need* in their jobs and daily lives. Unfortunately, his group believe that Algebra I is sufficient preparation for college and the work force. Thus the original Sept. 2009 draft of Core Standards only covered material through Algebra I. I, as a member of Validation, complained very strongly that this was nowhere near enough. But it amazed me how difficult it was to get anything else added, and all I was able to get was most of Algebra II. It was clear that the authors expected and probably wanted to include more advanced material including trigonometry and pre-calculus. But this did not happen.

3. What is a good way to "fight" back against the math so our children learn the basics?

   Work with your kids at home. Put them into solid private or charter schools that teach the basics. As a last resort, try to find solid tutors. It does not seem useful to try to convince the local public schools to change things. At best, if parents do succeed in getting the local schools to change their curricula, their own kids will have passed through before anything is changed. In my own case, back in the 1970's my kids were exposed to the New Math. I worked with them as much as I could and at least my son received a PhD in molecular biology with a minor in mathematics. Some parents who have taken these routes have expressed severe annoyance at

the schools taking advantage by including their kids scores in the school scores, but typically, it is possible to opt out of the tests.

4. Why do they skip over so many basics? What reason did they give for such poor standards?

   See the above remarks.

5. As a parent of an elementary-aged student, I see an overemphasis on word problems and an under-emphasis on practice of basic computations. This is troubling. Back in the "old days," mastering the basics -- facts and basic computation skills-- was done on school time. What are your thoughts on this?
   I agree, but maybe not for the same reasons. I believe that students need to get a lot further in mathematics these days than was typically the case for past generations. In particular, they need to be able to handle abstraction far better than we had to. However, to my knowledge, the best way to get to this level is to really learn the basics well. There is a tendency on the part of educators to say "if they need to learn e.g. problem solving, then we will simply teach it. However, mathematics is the most hierarchical of subjects. Solving advanced problems in mathematics only peripherally involves generalized strategies (if indeed there are any such things), and is crucially dependent on fully understanding everything that came before.

6. Are there parts of CCSS math standards that can be compressed or accelerated, so that students can get to pre-calc or calculus in high school with these standards?
   Of course, but the standards appear at specific grade levels in the lower grades, so this is where the tests will cover the material. In these circumstances, it is virtually impossible for teachers to effectively accelerate instruction.

7. Where does NCTM (National Council of Teachers of Mathematics) stand on the validity of these standards?

8. ***Why haven't the higher education colleges & Ivy League colleges weighed in more to advise what it truly necessary to enter stem careers or stem programs? Why is there not more outcry from them calling out the lack of content to truly prepare students?***
   Give them time. Right now, they are not at all interested in these issues. Moreover, at least with Stanford, if we are not satisfied with the quality of the top students, we will simply admit more students from the top Pacific Rim countries.

9. ***What are the critical concepts that must be taught? That seems to be excluded from common core standards??***
   I think that over the years, at least in the high schools we have achieved a pretty clear understanding of the key material students need. Though workforce requirements are changing the material that should be taught does not change. Rather it INCREASES. This is a particular consequence of the hierarchical nature of the subject. As an example, for more and more factory floor jobs now, students need to ADD coursework in computer programming.

10. Does he have any recommendations for parents that have gifted math children? How to supplement?? What kinds of things can parents do to help keep their kids pushing forward?
    There are specialized programs for gifted students now. For example the Stanford based EPGY (Education Program for Gifted Youth), and the EPGY Online High School. They are not free, but

they are very well tested and get very good results. And I imagine that there are a reasonable number of other choices along these lines.

11. Is there a source that tells parents what is considered "at grade level" ... From classical instruction with a true pathway to stem careers or programs? Basically looking for a guide to help us understand where our children, at minimum, should be.
See my remarks on question 16.

12. ***What were the repercussions for the kids in Russia when they had to pull back the failed geometry?? Looking to understand what we can expect. Is geometry truly moving towards the teaching method that failed in Russia?***

I don't know. I do know that the experiment only lasted a few years. My guess is that the teachers in the USSR, being pretty knowledgeable about the subject, and having a strong tradition in that area with the standard geometry text by Kiselev simply continued to use it.

13. ***What happens when the kids get to the later grades and can't work fractions because they do not know the basics? How hard is it to teach a child the basics that they needed as a foundation?***
If fractions are done right, following the actual suggestions in CC, then there is a reasonable expectation that the students will understand them pretty well. The problem comes up with the implementation. My guess is that the teachers will find it far too difficult to redo their own understandings so they can teach the CC procedures which are strongly modeled on the methods used in the high achieving countries, and nothing will change. The result will be that kids will be, as is typically the case today, virtually unable to understand or use fractions.

14. ***Were the common core math standards written by an English major who was mathematically challenged and the only way they could best understand math was to verbally explain it?***
There may well have been some of that going on.

15. What must be done before a student is ready for first semester college calculus?
Hard to say. One of the key things is that students have to understand the notion of slope very well indeed. Also, students should have some appreciation of what continuous means, and some idea of what limits are. Things that are very helpful are a solid understanding of the trigonometric functions, and the ability to graph and identify things like the vertex in graphs of quadratic functions in one variable.

16. What are the ramifications of developmentally inappropriate math curriculum on young children?
I find the term “developmentally inappropriate” very hard to pin down. Typically, I have heard it used by school officials when they explain why the students in their schools cannot handle material that is routinely learned by students in the high achieving countries at the grade level being discussed.

17. Did you have an opportunity to submit any math standards or recommendations and were they implemented?
I certainly had to strongly push for more algebra than was in the original draft. These recommendations and a number of others in the lower grades were implemented.

http://blogs.edweek.org/edweek/curriculum/Lowering%20the%20Bar.pdf

18. ***Can Dr. Milgram elaborate on his statement "It is extremely rare for students who begin their undergraduate years with coursework in precalculus or an even lower level of mathematical knowledge to achieve a bachelor's degree in a STEM area.18 Also, students whose last high school mathematics course was Algebra II have less than a 40 percent chance of obtaining a four- year college degree." From the recent White Paper of "Lowering the Bar" Sept. 2013***

Table 5. Bachelor's degree attainment rate by highest level of mathematics reached in high school by 1982 and 1992 12th-graders

	Class of 1982		Class of 1992	
Level of math	Percentage reaching this level of math	Earned bachelor's	Percentage reaching this level of math	Earned bachelor's
Calculus	5.2 (0.36)	82.1 (2.45)	9.7 (0.54)	83.3 (2.72)
Precalculus	4.8 (0.37)	75.9 (2.43)	10.8 (0.65)	74.6 (2.04)
Trigonometry	9.3 (0.51)	64.7 (2.32)	12.1 (0.81)	60.0 (3.32)
Algebra 2	24.6 (0.75)	46.4 (1.54)	30.0 (1.08)	39.3 (2.31)
Geometry	16.3 (0.65)	31.0 (1.92)	14.2 (0.87)	16.7 (1.87)
Algebra 1	21.8 (0.69)	13.4 (1.33)	16.5 (0.92)	7.0 (1.24)
Pre-algebra	18.0 (0.66)	5.4 (1.19)	6.7 (0.53)	3.9 (1.34)

NOTES: Standard errors are in parentheses. The columns for level of math may not add to 100.0 percent due to rounding.

SOURCES: National Center for Education Statistics: High School & Beyond/Sophomore Cohort (NCES 2000-194) and NELS:88/2000 Postsecondary Transcript Files (NCES 2003-402 and Supplement).

Data from the NCES. Also NCES is the source for the following table showing the odds of completing a STEM degree based on the level of the first math course taken:

National Center for Education Statistics

Table 7.
HIGHEST MATH COURSE IN FIRST YEAR: Percentage distribution of the highest level of mathematics in which 2003–04 beginning bachelor's and associate's degree students earned credits, by STEM entrance and persistence through 2009

	Beginning bachelor's degree students				Beginning associate's degree students			
STEM entrance and persistence through 2009	No math	Precollege-level math only[1]	Introductory college-level math[2]	Calculus and advanced math	No math	Precollege-level math only[1]	Introductory college-level math[2]	Calculus and advanced math
Total	40.1	8.7	30.1	21.2	49.2	24.5	22.9	3.4
Students who entered STEM fields in first year								
STEM leavers[3]	34.3	9.3	24.0	32.4	44.2	21.2	28.4	6.2
Students who left PSE without a degree/certificate	39.9	12.1	20.2	27.8	50.5	16.2	28.0	5.3 !
Students who switched major to a non-STEM field	29.7	7.0	27.1	36.2	36.8	27.1	28.9	7.2 !
STEM persisters/completers	14.3	3.1 !	19.3	63.3	25.1	13.9	33.4	27.6
Students who completed a STEM degree/certificate	13.7	2.1 !	15.0	69.2	16.8 !	12.2 !	44.0	27.1
Students who entered STEM fields after first year								
STEM leavers[3]	36.4	10.7	30.1	22.8	43.5	22.5	30.6	3.3 !
Students who left PSE without a degree/certificate	34.6	11.4 !	36.1	18.0	48.9	27.9	19.7	‡
Students who switched major to a non-STEM field	37.6	10.3 !	26.4	25.7	35.8	14.9 !	46.3	‡
STEM persisters/completers	27.1	5.4	20.0	47.6	37.5	17.8	27.4	17.3
Students who completed a STEM degree/certificate	24.2	4.3 !	17.4	54.1	18.5 !	12.6 !	37.0	31.9 !

See notes at end of table.

These are part of the data on U.S. Education outcomes collected and kept reasonably current by the National Center for Education Statistics, a department in the U.S. Department of Education.

19. Could he please elaborate more on the place-markers (commonly called stubs) for standards that the writers intended to fill in later?

There isn't much to say. Here, scanned from the March 10, 2010 CC Math Standards draft, is the set of stubs:

Limits and Continuity†	F-LC
Differential Calculus†	F-DC
Applications of Derivatives†	F-AD
Integral Calculus†	F-IC
Applications of Integration†	F-AI
Infinite Series†	F-IS

It should be pretty clear that these list the basic topics in a reasonable calculus course. From the form and placement of this list it seemed pretty clear that the authors had every intention of filling in details and including more advanced (probably (+)) standards in the document. However, for some reason the final document contained no such standards or even stubs like the above.

20. ***Do the standards really end at Algebra II/ Trigonometry? Is Pre-Calculus ever going to be built in to the CCSS?***

There are no current plans to change the CCSSI Math Standards.

21. Can you explain how CC pushes algebra 1 to ninth grade?

In almost all the states, Algebra I is a ninth grade class, though students can often test into an Algebra I course a year or two earlier. Thus, it was the other way round. If the authors of CCSSI had written in a pathway to Algebra I in seventh or eighth grade, it would have been regarded as tracking, and the state buy-in would have been very much reduced. Likewise, if they had arranged things so that every student would take Algebra I in grade 8, they would have had very few states willing to sign on, even with the 4.5 billion dollar inducement of RttT.

22. How does CC diminish STEM education?

It doesn't. It's just that when one does not require high schools to give more than three years of mathematics culminating in Algebra II, then the chances of a district offering more advanced courses like linear algebra, trigonometry, precalculus and calculus become much more dependent on the average wealth of the district. As a result, there is a de facto lowering of the opportunities for students coming from less wealthy districts, particularly in STEM areas. Here is

Table 6. Percentage of 1992 12th-graders who attended high schools that offered courses[a] in statistics, trigonometry, and calculus, by socioeconomic status quintile

	Percent attending high schools that offered:		
Demographic group	Calculus	Trigonometry	Statistics
Socioeconomic status quintile			
Highest quintile	71.6 (1.93)	83.1 (1.64)	34.0 (2.30)
2nd quintile	56.2 (2.32)	73.2 (2.13)	27.1 (2.01)
3rd quintile	54.1 (2.39)	71.4 (2.33)	24.9 (1.92)
4th quintile	49.3 (2.46)	70.3 (2.28)	20.3 (1.80)
Lowest quintile	43.5 (2.86)	63.7 (2.66)	18.5 (2.06)

[a] Responses are based on surveys of school administrators and math teachers of NELS students in 1990. Where the administrator did not answer the question, the math teachers did not indicate that they taught the subject, and students did not earn any credits in the subject, the calculation assumes that the school did not offer the subject. This approach may underestimate the percentage of high school offering the subjects at issue.
NOTES: Standard errors are in parentheses.
SOURCES: National Center for Education Statistics: NELS:88/94 (NCES 96-130), and NELS:88/2000 Postsecondary Transcript Files (NCES 2003-402).

the actual data from the 1990's:

23. If students can't make it to Calc by twelfth grade, how does that impact their choice of colleges?

For the most part, it is pre-calculus that matters. Of course, it is more and more the case at the elite schools like Stanford that calculus is regarded as somewhat remedial. For example, the course with the largest single enrollment at Stanford is the Fall Quarter advanced calculus course, Math 51. By the end of their freshman year, over 2/3 of the students will have had at least one quarter of advanced calculus.

24. Does the new method for teaching Algebra 1 have a proven track record?
I don't really know if it is a new method. My impression was that it is pretty standard.

25. ***As a parent of a child in an elementary school/primary grade, I'd like to know what your research & experience tells us about how kids learn math: is there a "set way" most kids develop their number sense and reasoning, or do children approach mathematics in a variety of ways depending on their background and learning style? Is there one "right" way to do math?***
There is a sequence of things kids have to learn, first basic numeration, counting objects, knowing the number words in our language, second abstracting the concept of numbers (knowing that numbers are objects that one can add and multiply, and the basic laws – associativity, commutativity, distributive rule, and cancellation). In this country, the last is considered too difficult and is not done. But in the high achieving countries it is the real focus of the first two years of instruction. Then, with this background, kids expand on the numbers they know to include fractions which are also numbers. It isn't important "how" one learns this material, but the ordering appears natural. Students in these high achieving countries have very high high school graduation rates – generally over 90% – and somewhere between 50 and 100% of the graduates will have had a calculus course.

26. What role do "math facts" play in the development of mathematical understanding and a child's ability to learn or master a new mathematical process? I was once under the impression that the ability to recall math facts quickly and accurately will assist a child in her understanding of processes. For instance, a child must know his basic multiplication facts (5x7=35) before he can master more 'difficult' problems/processes such as 158 x 7. Is this assumption valid? Also, should parents be helping kids with the rote memorization of math facts? If so, which facts, at what age? Additionally, in what format? Numerically or by using words? (I do understand that all children all different)
They are crucial. There are two things that students need to have to continue with mathematics. The first is fluency with the basic operations of addition and multiplication, as well as some degree of comfort with the inverse operations of subtraction and division. The simplest and best way of doing this is for students to become fluent with the standard algorithms. The second is an understanding of why (base 10 notation and the basic laws satisfied by the operations, distributive, associative, and commutative) these standard algorithms actually work. This second can be somewhat informal, but it had better be there. However, it is most likely that students will not be able to achieve this "why" understanding if they are not fluent with the standard algorithms.

27. ***Many parents are seeing a new way to teach math in the early grades that only seems to make the students frustrated and lose interest. Based on your expertise, is the approach being taught under Common Core at the elementary level appropriate?***
This is actually interesting. Common Core does not suggest the approaches that are being used, but when the teachers and, above all, their supervisors and instructors, do not understand the mathematics that is actually being presented in CC, they seem to interpret the CC standards to

mean things the authors did not expect. Basically, it seems that they focus on the "Standards for Mathematical Practice" and interpret these as though they were the old 1989 NCTM standards, so all the old verbiage from the 1990's is repeated in Professional Development sessions for CC, and the programs that we got rid of nearly 20 years ago are being readopted by the schools.

28. ***I hear many parents and students struggling with the Common Core math as it is being currently taught (i.e. the math is too hard) and then I also hear that you do not believe students finish high school with enough math (i.e. the math is not hard enough). Can you explain the contradiction? If the Common Core standards were as advanced as required to achieve the level you believe they should have at the end of high school (to be competitive with other countries), would the average early elementary student be able to do the required math?***

It's not a contradiction. The early grade math is not hard at all, but when the elementary school teachers have been able to obtain certifications without ever taking a single college math course (or – if a college math course is actually a requirement -- taking a super-simplified course that I have heard described as "math for vegetables") then the teachers will simply be unable to handle the material in the early grades that CC expects will be delivered. And when the teachers do not understand what is being asked, the students will find the material hard. However, the situation is quite different in the upper grades where deliberate choices have been made to cut off mathematics instruction after only geometry and algebra II, since this is the minimum level that gives students about a 1 in 3 chance of completing a college degree. (By contrast, if the cut-off had been Algebra I, then the chance of obtaining a college degree would have been about 1 in 14.) This is a huge contrast with what is done in the high achieving countries.

29. If the cc math is so poor that you will end up learning less than before (i.e.: no pre-calculus) and you won't have the math needed for admittance to a 4 yr. university, then why the is it so confusing and hard?

See my comments above. Hardness is a function of many things. One of them is the understanding of the topics being delivered by the instructors. If they do not understand the material, and they typically don't, then the students will be completely confused and everything will appear incomprehensible, hence "hard."

30. ***It seems as if probability and mean mode median and range are missing in grades k-5 then in 6th the students are not ready for the statistics because they are not prepared for those concepts because they did not learn the foundational concepts in the elementary grades. Is this so?***

Not quite. If the students are taught these topics in a reasonable manner and they have actually learned what was expected and hoped for by CC's authors in K-5, then they will have very little trouble. The instruction process should be (1) CAREFULLY DEFINE MEAN, MODE MEDIAN, AND RANGE. (2) lots of examples to show how to caculate them in particular paying close attention to the (1/n) in the definition of the median.

31. What do you believe is the ultimate goal for Common Core? As is typically the case with documents written by large and diverse committees, there are many different goals in the mix, and some of them are not even compatible with the others

32. In the past, what roles have been represented in the writing of valid K-12 standards? (Ex: child development experts? subject matter, etc.) In the highest achieving countries, the dominant role has been played by (actual) mathematicians and math educators who have prepared for their

specific specialty by first taking every basic graduate course in mathematics that every other student in math is expected to learn. These people have been typically advised by top level cognitive psychologist and teachers at all grade levels, but the final decisions on content have been theirs. This is not the case at all in this country.